Robust Communication Connectivity for Multi-Robot Path Coordination using Mixed Integer Nonlinear Programming: Formulation and Feasibility Analysis

Pramod Abichandani, Hande Y. Benson, and Moshe Kam

Abstract— **Mixed Integer Nonlinear Programming (MINLP) techniques are increasingly used to address challenging problems in robotics, especially Multi-Vehicle Motion Planning (MVMP). A particular challenge in using this framework is encoding stochastic phenomena such as communication connectivity in the form of MINLP constraints.**

The main contribution of this paper is an analytical formulation of communication connectivity constraints using stochastic physical layer communication models. These constraints account for the log-normal channel shadowing in noisy communication environments and specify inter-vehicle connectivity in terms of the outage probability of communication. A method is developed to provably accord robustness to communication failure by specifying an upper bound on the outage probability in terms of the inter-vehicle communication range.

Finally, we demonstrate the utility of this formulation in the context of a realistic decentralized Multi-Vehicle Path Coordination (MVPC) scenario in which multiple robotic vehicles travel along predetermined fixed paths and are required to maintain communication connectivity during their transit. Conditions that affect the feasibility of the MVPC problem are formaliz1ed. Examples that assist in visualizing these conditions are provided.

I. INTRODUCTION

Mathematical Programming (MP) techniques are increasingly used to address challenging problems in Multi-Vehicle Motion Planning (MVMP) [1]. In recent work, we reported on the results of our experiments in real-time Mixed Integer Nonlinear Programming (MINLP) based decentralized Multi-Vehicle Path Coordination (MVPC) under communication connectivity constraints [2]. These experiments involved a group of 5 robotic vehicles moving along pre-determined and fixed paths while avoiding collisions and maintaining communication connectivity with the other robots.

Only a handful of studies have documented practical implementations of MP-based motion planning (e.g. [3], [4]) and to the best of our knowledge, our work in [2] presented the first experimental results for MVPC under communication connectivity constraints using MINLP. The formulations in [2] were deterministic in nature in that they did not account for the uncertainties in inter-vehicle communication.

In this paper, we extend this work by incorporating stochastic physical layer communication models into the formulation of inter-vehicle communication connectivity requirements. Specifically, we account for shadowing effects in wireless communication by using an established log-normal model [5]. Inter-vehicle communication connectivity is specified in terms of outage probability of communication i.e., the probability that the Signal-to-Noise (SNR) ratio experienced by a transceiver is less than a specified threshold.

Robustness is accorded by bounding the outage probability from above, resulting in a set of probabilistic constraints that specify inter-vehicle communication connectivity. In order to make this probabilistic formulation amenable to MINLP computations, we transform it to an equivalent constraint on inter-vehicle communication range. Feasibility conditions that affect the communication-centric requirements of this problem are formalized and presented along with visual illustrations.

While several approaches to robust motion planning have been reported in the literature, to the best of our knowledge, this paper presents the first MINLP formulations for the MVPC problem [6], [7]. The MVPC problem is relevant in situations where one does not get the liberty of planning arbitrary paths, and rather, must follow a prescribed route. Examples include multiple driverless car-like vehicles operating in an urban environment. The formulations provided in this paper address the communication-centric motion planning challenges associated with such unmanned operations.

II. MVPC MODEL ELEMENTS

There are three basic elements of this model: robot architecture and motion, robot paths, and inter-robot communication.

A. Robot Architecture

Consider a group of n two-wheeled differential drive mobile robots as shown in Figure 1. These robots move in a global Cartesian co-ordinate plane with non-holonomic constraints that disallows them from sliding sideways. s and ω are the linear and angular speeds of the robots respectively. x, y and θ are the coordinates of the robots with respect to the global coordinate system. Each robot $i = 1,\ldots,n$ has a fixed path p^i to follow, with a given start (origin) point o^i and a given end (goal) point e^i. P is the set of all the fixed

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paths of each robot. $p^i \in P, i = 1,\ldots,n$. O is the set of all start (origin) points. $o^i \in O, i = 1,\ldots,n$. E is the set of all end points. $e^i \in E, i = 1,\ldots,n$. The Euclidean distance between two robots i and j is denoted by d^{ij}. The robots are required to maintain a minimum safe distance d_{safe} in order to avoid collisions with each other. The distance between the current location and the goal point for robot i is given by d^i_{goal}. The speed of the robot i along its fixed path is denoted by s^i.

B. Robot Paths

Each robot follows a fixed path represented by a two-dimensional piecewise cubic spline curve. The curve is obtained by combining two one-dimensional piecewise cubic splines, $px(u)$ and $py(u)$, where the parameter u is the arc length along the curve. These piecewise cubic splines have continuous first derivatives (slope) and second derivatives (curvature) along the curve. The paths represented by the two-dimensional piecewise cubic splines, along with the constraints on the speed, acceleration and turn rate result in a kinodynamically feasible trajectory. For a detailed discussion on spline curve design and analysis, readers are referred to [8] and its references.

C. Communication Model

Each robot is equipped with a wireless transceiver. The communication constraint requires that at all times, every robot is in communication range of at least n_{conn} other robots, where n_{conn} varies between 0 and $n-1$. The Signal to Noise Ratio (SNR) experienced by the receiver robot is calculated to determine whether or not the communication constraint is satisfied or not. If the SNR experienced by a receiver node placed on a robot is above a predefined threshold η_c, the two robots are considered to be in one-hop communication range of each other.

Consider two robots i and j that try to communicate with each other at a given point in time. The Euclidean distance between them is denoted by d^{ij}. The signal transmission power of the wireless transceiver placed on the transmitter robot is denoted by P^i_t. The received signal power of the wireless transceiver placed on the receiver robot is denoted by P^j_r.

The power experienced by the receiver robot is calculated using Friis's equation [9]

$$P^j_r = P^i_t G_t G_r \left(\frac{\lambda}{4\pi d^{ij}}\right)^{\alpha} \tag{1}$$

where α is the path loss exponent. The values of α range from 1.6 (indoor with line of sight) to 6 (outdoor obstructed) depending on the environment. λ is the wavelength and is equal to c/f, where c = 3 x 10^8 m/s and f = 2.4 x 10^9 Hz. The values of G_t and G_r (antenna gains) are assumed to be 1.

Using (1), the average large-scale path loss between a transmitter receiver pair can be expressed as a function of distance:

$$\overline{PL}^{ij}(d^{ij}) \propto \left(\frac{d^{ij}}{d_0}\right)^{\alpha}. \tag{2}$$

When expressed in decibels (dB), (2) can be rewritten as

$$\overline{PL}^{ij}(d^{ij}) = \overline{PL}^{ij}(d_0) + 10\alpha \log\left(\frac{d^{ij}}{d_0}\right). \tag{3}$$

In (2) and (3), d_0 is the close-in reference distance that is in the far field of the antenna so that near-field effects do not alter the reference path loss. d_0 is calculated using (1) such that $d \geq d_0 \geq d_f$, where d_f is the far-field distance. d_f is calculated as

$$d_f = \frac{2D^2_{dim}}{\lambda}, \tag{4}$$

where D_{dim} is the largest physical linear dimension of the antenna. $d_f \gg D_{dim}$ and $d_f \gg \lambda$ for far-field regions. The bars in (2) and (3) denote the ensemble average of all possible path loss values for a given value of d.

In addition to the path-loss, shadowing effects due to the presence of obstacles in the environment cause random attenuation in wireless communication. Shadowing effects are represented by a log-normal random variable that is factored into (2) as indicated by

$$\overline{PL}^{ij}(d^{ij}) \propto \left(\frac{d^{ij}}{d_0}\right)^{\alpha} \Upsilon, \tag{5}$$

where Υ has a pdf

$$f_\Upsilon(x) = \frac{10/\ln 10}{\sqrt{2\pi}\upsilon x} exp\left\{-\frac{(10\log_{10}x)^2}{2\upsilon^2}\right\}, x \geq 0. \tag{6}$$

When expressed in decibels (dB), the path-loss can be calculated as

$$PL^{ij}(d^{ij})[dB] = \overline{PL}^{ij}(d_0) + 10\alpha \log\left(\frac{d}{d_0}\right) + \Upsilon_\upsilon, \tag{7}$$

where Υ_υ is the zero-mean Gaussian random variable with standard deviation υ. The received power $P^j_r(d)$ is expressed as

$$P^j_r(d^{ij})[dBm] = P^i_t[dBm] - PL^{ij}(d^{ij})[dB]. \tag{8}$$

The Signal to Noise Ratio (SNR) experienced by the receiver robot is calculated using the relationship

$$SNR^{ij} = P^j_r(d^{ij})/N, \tag{9}$$

where N is the noise experienced by receiver robot. The noise is assumed to be thermal $N = kTBF$, where k is the Boltzmann's constant given by 1.38×10^{-23} Joules/Kelvin, B is the equivalent bandwidth of the receiver, T is the ambient room temperature (typically 290 Kelvin to 300 Kelvin), and F is noise figure of the receiver.

The *outage probability* $\mathbb{P}^{ij}_0$ is defined as the probability that the SNR experienced is less than a certain threshold η_{snr}, or equivalently the received power P^j_r is below a certain threshold η_c:

$$\mathbb{P}_0^{ij} = \mathbb{P}(\text{SNR}^{ij} < \eta_{snr}) = \mathbb{P}(P_r^{ij} < \eta_c) \tag{10}$$

If the outage probability of communication $\mathbb{P}_0^{ij}$ for transmitter and receiver robot pair *i-j* is below a threshold η_ε, the the robots are considered to be in one-hop communication range of each other.

$$\begin{aligned} &\mathbb{P}_0^{ij} \leq \eta_\varepsilon \\ \Longleftrightarrow &\mathbb{P}(\text{SNR}^{ij} < \eta_{snr}) \leq \eta_\varepsilon \\ \Longleftrightarrow &\mathbb{P}(P_r^j(d^{ij}) < \eta_c) \leq \eta_\varepsilon \end{aligned} \tag{11}$$

Interested readers are referred to [5] and its references for further information about physical layer communication models.

III. Robust Communication Constraint Formulation

The probabilistic constraint in (11) represents inter-vehicle communication connectivity. This constraint can be transformed into an equivalent constraint on inter-vehicle communication range.

$$\mathbb{P}(\text{SNR}^{ij} < \eta_{snr}) = \mathbb{P}(P_r^j(d^{ij}) < \eta_c) \tag{12}$$

$$= Q\left(\frac{\overline{P_r(d^{ij})} - \eta_c}{\upsilon}\right), \tag{13}$$

where $Q(\cdot)$ is the Q-function represented as

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-\frac{x^2}{2}}\, dx. \tag{14}$$

For a given outage probability threshold for the transmitter receiver robot pair $i-j$, η_ε, the condition $\mathbb{P}_0^{ij} \leq \eta_\varepsilon$ leads to the following:

$$\begin{aligned} \mathbb{P}_0^{ij} \leq \eta_\varepsilon &\Longleftrightarrow \mathbb{P}(P_r^j(d^{ij}) < \eta_c) \leq \eta_\varepsilon \\ &\Longleftrightarrow Q\left(\frac{\overline{P_r(d^{ij})} - \eta_c}{\upsilon}\right) \leq \eta_\varepsilon \\ &\Longleftrightarrow \overline{P_r(d^{ij})} \leq \eta_c + \upsilon Q^{-1}(\eta_\varepsilon) \\ &\Longleftrightarrow P_t^i[dBm] - \overline{PL}^i j(d_0) - 10\alpha \log \frac{d^{ij}}{d_0} \leq \eta_c + \upsilon Q^{-1}(\eta_\varepsilon) \\ &\vdots \\ d^{ij} \leq 10\hat{}&\left(\frac{\eta_c + \upsilon Q^{-1}(\eta_\varepsilon) - P_t^i[dBm] + \overline{PL}^{ij}(d_0) - 10\alpha\log(d_0)}{-10\alpha}\right) \end{aligned} \tag{15, 16}$$

Thus, the condition $\mathbb{P}_0^{ij} \leq \eta_\varepsilon$ leads to the condition $d^{ij} \leq \eta_{rd}$, where η_{rd} is given by right-hand side of the inequality (16).

We introduce a binary variable $C^{ij}(t)$ that indicates whether the two robots are in one-hop communication range of each other or not. $C^{ij}(t) = 1$ for the robots i and j if they are in one-hop communication range of each other at time t, i.e., $\mathbb{P}(SNR^{ij}(t) \leq \eta_{snr}) \leq \eta_\varepsilon$ and $C^{ij}(t) = 0$ if $\mathbb{P}(SNR^{ij}(t) \leq \eta_{snr}) > \eta_\varepsilon$. Then the communication constraints can be expressed by (17) and (18):

$$\sum_{j:j\neq i} C^{ij}(t) \geq n_{conn} \tag{17}$$

$$C^{ij}(t) = \begin{cases} 1, & \text{if } d^{ij} \leq \eta_{rd}, \\ 0, & \text{otherwise.} \end{cases} \tag{18}$$

The constraint (17) indicates the fact that at any given point in time, each robot should be in one-hop communication connectivity with at least n_{conn} other robots. The disjunctive one-hop communication constraints (17) and (18) are reformulated as Big-M constraints given by (19)-(21) [10]. For this reformulation, we introduce a constant M_1 and formulate constraint (20). In the case $d^{ij}(t) \leq \eta_{rd}$, $C^{ij}(t)$ can assume a value of either 0 or 1. But the constraint (21) forces at least n_{conn} of them to be set to 1. This means that only n_{conn} of the C^{ij} variables, and not necessarily all, are guaranteed to have the correct value $C^{ij} = 1$. If $d^{ij} > \eta_{rd}$, then $C^{ij}(t)$ will have to equal 0 in order to satisfy (20).

$$C^{ij}(t) \in \{0,1\} \tag{19}$$

$$d^{ij}(t) \leq M(1 - C^{ij}(t)) + \eta_{rd} \tag{20}$$

$$\sum_{j:j\neq i} C^{ij}(t) \geq n_{conn} \tag{21}$$

IV. Decentralized MVPC Model Formulation

We provide an abbreviated discussion of the decentralized MVPC model that was first presented in [11] and experimentally validated in [2].

A. Receding Horizon Planning

The parameter t represents steps in time. T_{hor} is the receding horizon time. At each time step t, each robot must calculate its plan for the next T_{hor} time steps, and communicate this plan with other robots in the network. While the robots compute their trajectory points and corresponding input commands for the next T_{hor} time steps, only the first of these solutions is implemented, and the process is repeated at each time step. T_{max} is the time taken by the last arriving robot to reach its end point. At $t = T_{max}$ the scenario is completed. If a robot reaches the goal point before T_{max}, it continues to stay there until the mission is over. However, if required, it can still communicate with other robots.

Each robot plans its own trajectory by taking the plans of all other robots into account at each discrete time step. For a given time step t, each robot determines its speed for the next T_{hor} time steps starting at time t i.e. $s^i(t), \ldots, s^i(t+T_{hor})$ and implements the first speed $s^i(t+1)$ out of all these speeds. In this way, the plan starting at time step $t+1$ must be computed during time step t. Thus, during each time step t, each robot communicates the following information about its plan $\mathscr{P}^i(t)$ to other robots: $\mathscr{P}^i(t) = [\mathbf{p}^i(t) \ldots \mathbf{p}^i(t+T_{hor})]$, where $\mathbf{p}^i(t) = (x^i(t), y^i(t))$ is the location of the robot i on its path at time t calculated based on the optimal speed $s^i(t)$.

B. Decision Ordering

The robots are assigned a pre-determined randomized decision order. The decentralized algorithm presented in [2] sequentially cycles through each robot, thereby allowing each robot to solve its planning problem in the order $ord(i)$, $i \in \{1,\ldots,N\}$. Each robot i solves the optimization problem $\mathscr{O}^i(t)$ indicated by (22)-(34) in the order $ord(i)$ that it has been assigned:

$$\text{minimize} \quad \sum_{k=t}^{t+T_{hor}} d^i_{goal}(k) \tag{22}$$

$$\text{subject to} \quad \forall j \in \{1,\ldots,N\}, j \neq i$$

$$\forall k \in \{t,\ldots,t+T_{hor}\}$$

$$(x^i(0), y^i(0)) = o^i \tag{23}$$

$$u^i(0) = 0 \tag{24}$$

$$u^i(k) \leq U^i \tag{25}$$

$$u^i(k) = u^i(k-1) + s^i(k)\Delta t \tag{26}$$

$$(x^i(k), y^i(k)) = ps^i(u^i(k)) \tag{27}$$

$$s_{min} \leq s^i(k) \leq s_{max} \tag{28}$$

$$a_{min} \leq a^i(k) \leq a_{max} \tag{29}$$

$$d^i_{goal}(k) = U^i - u^i(k) \tag{30}$$

$$d^{ij}(k) \geq d_{safe} \tag{31}$$

$$d^{ij}(k) \leq M(1 - C^{ij}(k)) + \eta_{rd} \tag{32}$$

$$\sum_{j:j\neq i} C^{ij}(k) \geq n_{conn} \tag{33}$$

$$C^{ij}(k) \in \{0,1\}. \tag{34}$$

C. Decision Variables

In (22)-(34), the independent decision variables are the speeds, $s^i(t),\ldots,s^i(t+T_{hor})$, for robot i at time t. The values of the remaining variables are dependent on the speeds.

D. Objective Function

Equation (22) represents the objective function to be minimized. This formulation forces the robots to minimize the total distance between their current location and the goal position over the entire receding horizon. Constraint (30) defines the distance to goal $d^i_{goal}(k)$ for each robot $i = 1\ldots N$ at time-step k, $\forall k \in \{t,\ldots,t+T_{hor}\}$ as the difference between its path length U^i and the total arc length travelled $u^i(k)$. The choice of this objective function results in the robots not stalling and moving to their goal position as fast as possible (minimum time solution).

E. Path (Kinematic) Constraints

Constraints (23)-(27) define the path of each robot. The constraints (23), (24), and (25) form the boundary conditions. Constraint (23) indicates that each robot i has to start at a designated start point o^i. Constraint (24) initializes the arc length travelled u to zero value. Constraint (25) provides the upper bound on the arc length travelled. Constraint (26) increments the arc length at each time step based on the speed of the robot ($\Delta t = 1$). Constraint (27) ensures that the robots follow their respective paths as defined by the cubic splines. The function $ps^i(u^i(k))$ denotes the location of robot i at time step k, $\forall k \in \{t,\ldots,t+T_{hor}\}$ after travelling an arc length of $u^i(k)$ along the piecewise cubic spline curves. It should be noted that constraint (27) is a nonlinear equality constraint, which means that $\mathscr{O}^i(t)$ is a mixed integer nonlinear programming problem (MINLP) with a non-convex nonlinear relaxation.

F. Speed and Acceleration (Dynamic) Constraint

Constraints (28)-(29) are dynamic constraints and ensure that the speed $s^i(k)$ (and hence, angular velocity) and the acceleration $a^i(k)$ for each robot $i = 1,\ldots,N$ at each time-step k, $\forall k \in \{t,\ldots,t+T_{hor}\}$ are bounded from above (by s_{max} and a_{max}, respectively), and below (by s_{min} and a_{min} respectively). These constraints are determined by the capabilities of the robot and the curvature $\kappa(u)$ of the paths represented by the spline curve. Here we assume that the curvature of the paths is within the achievable bounds of the angular speed and radial acceleration of the robots. Hence the angular speed required by the robots corresponding to the optimal speed is always achievable.

G. Collision Avoidance Constraint

The constraint (31) ensures that there is a sufficiently large distance d_{safe} between each pair of robots to avoid a collision at all times. In addition to constraint (27), constraint (31) is another reason why $\mathscr{O}^i(t)$ has a relaxation that is a non-convex nonlinear programming problem.

H. Communication Connectivity Constraint

As discussed in Section III, constraints (33) and (34) state that vehicle i should be in communication range of at least n_{conn} vehicles. This means that, for at least n_{conn} values of $j = 1,\ldots,N$, $j \neq i$, the condition $d^{ij}(k) \leq \eta_{rd}$ should be satisfied. The remaining vehicles may or may not be in communication range of i.

In our numerical implementation in [2], [11], we used an equivalent form of constraint (20) as (35):

$$\frac{(d^{ij}(t))^2}{(M(1-C^{ij}(t)) + \eta_{rd})} \leq (M(1-C^{ij}(t)) + \eta_{rd}) \tag{35}$$

This form is chosen in order to avoid the nondifferentiability of a Euclidean distance calculation within the nonlinear solver MILANO [12], [13]. The nondifferentiability is not going to occur at the optimal solution due to the collision avoidance requirements keeping d^{ij} sufficiently large, but during the initial iterations of the MILANO solver, it may cause numerical difficulties. The reformulation removes the

potential of such an occurrence and provides numerical stability.

I. Remarks about the quality of solutions

As mentioned in [11], it was found that the decision order $ord(i)$ of the robots $i = 1,\ldots,n$ can qualitatively affect the solution of each robot depending on the geometry of the paths. Due to the inherent decentralized decision making, certain robots' decisions may render the coordination problem difficult to solve for other robots. In some cases, reassigning a different decision order $ord(i)$ of the robots $i = 1,\ldots,n$ helped improve overall solutions. Also in some cases, certain robots' decisions can render the coordination problem infeasible for other robots regardless of the decision ordering used. In such cases, the robots may use their plans from the previous time steps given the receding horizon nature of this approach.

V. Feasibility Considerations

Several conditions that will result in feasibility of the MINLP (22)-(34) presented in Section IV are identified and formalized. We start by formalizing the notion of feasible configurations and noting Lemma 2 which states that the communication constraints are the only source of infeasibility for (22)-(34).

Lemma 1: The collision avoidance distance d_{safe} should be sufficiently less than the maximum separation between any two robots to not cause infeasibility in (22)-(34).

Proof: If Lemma 1 is not satisfied, then there can be a case as shown in Figure 1. It is trivial to see that this lemma is critical as it enforces a conservative limit on the value of d_{safe} such that, at all times, it is sufficiently less than the maximum separation between any two robots. ■

Definition 1: **(Feasible configuration)** *Define a feasible configuration $\mathscr{F}(t)$ at discrete time-step t as the set of locations $(x^i(t), y^i(t))$ along the path p^i for each robot i such that no robots collide with each other, i.e.,*

$$\mathscr{F}(t) = \{(x^i(t), y^i(t)), \forall i, j \in 1,\ldots,n, j \neq i |\ d^{ij}(t) \geq d_{safe}\}. \quad (36)$$

Lemma 2: Given a set of n robots with fixed paths whose start (origin) and end (destination) points are feasible configurations, in absence of the communication constraints, the MINLP (22)-(34) *will always be feasible if Lemma 1 holds.*

Proof: The communication connectivity constraints restrict the robots to move such that they remain in communication range of each other. In absence of such constraints, a trivial feasible solution for (22)-(34) is always available wherein the robots move one after the other such that only one robot moves at a time and the other robots wait till the robot in transit reaches its destination. ■

Definition 2: **(Infeasible Paths)** *The set of paths P is defined as infeasible if for some path $p^i \in P$ of robot i, there exists a closed interval of arc-lengths $[u^i_{infe_start}, u^i_{infe_end}]$ such that $\forall u^i_{infe} \in [u^i_{infe_start}, u^i_{infe_end}]$ condition (37) is satisfied.*

$$\sqrt{(x^i(u^i_{infe}) - x^j(u^j))^2 + (y^i(u^i_{infe}) - y^j(u^j))^2} > \eta_{rd}, \quad (37)$$

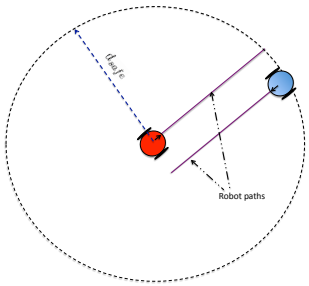

Fig. 1. The radius of the dotted circle represents d_{safe}. The two robots are at their starting points. It is clear that with this d_{safe} value, the problem is infeasible.

$\forall u^j \in [0, U^j]$, $\forall j \in \mathscr{Y}$. The set $\mathscr{Y} \subset \{1,2,\ldots,n\}$ with cardinality $|\mathscr{Y}| > n - n_{conn} - 1$.

Infeasible paths are characterized by the fact that the physical separations between them can be larger than inter-robot communication ranges. An example of *Infeasible Paths* is described in Figure 2. This set is an example of a case where the communication constraints are not satisfied due to path limitations. This condition is sufficient for (22)-(34) to be infeasible.

Definition 3: **(Infeasible Set)** *The set of paths P is defined as an infeasible set if for some closed interval of arc-lengths $[u^i_{dinfe_start}, u^i_{dinfe_end}]$ along the path $p^i \in P$ for robot i, there exists some closed interval of arc-lengths $[u^j_{dinfe_start}, u^j_{dinfe_end}]$ along the path $p^j \in P$ for some robot $j \in \mathscr{Y}$, where the set $\mathscr{Y} \subset \{1,2,\ldots,n\}$ with cardinality $|\mathscr{Y}| > n - n_{conn} - 1$, such that the robot j has to move at infinite speed to satisfy the condition (38)*

$$\sqrt{(x^i(u^i_{dinfe}(t)) - x^j(u^j_{dinfe}(t)))^2 + (y^i(u^i_{dinfe}(t)) - y^j(u^j_{dinfe}(t)))^2} \leq \eta_{rd}. \quad (38)$$

An infeasible set is shown in the Figure 3. The figure to the left shows the initial feasible configuration of five (5) robots with $n_{conn} = 2$. The two dotted circles indicate the communication range of the red and yellow robots, respectively. In this configuration, all robots are in communication

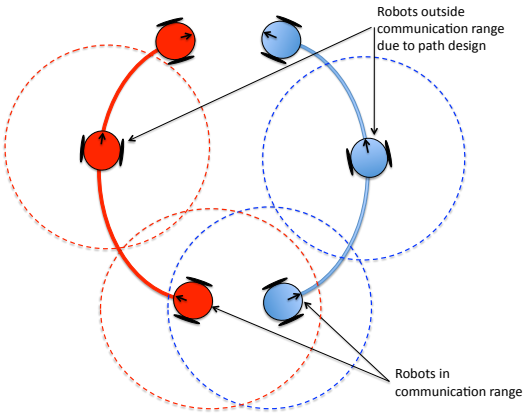

Fig. 2. The paths shown here are *Infeasible Paths*. The dotted circles indicate the communication range of each robot.

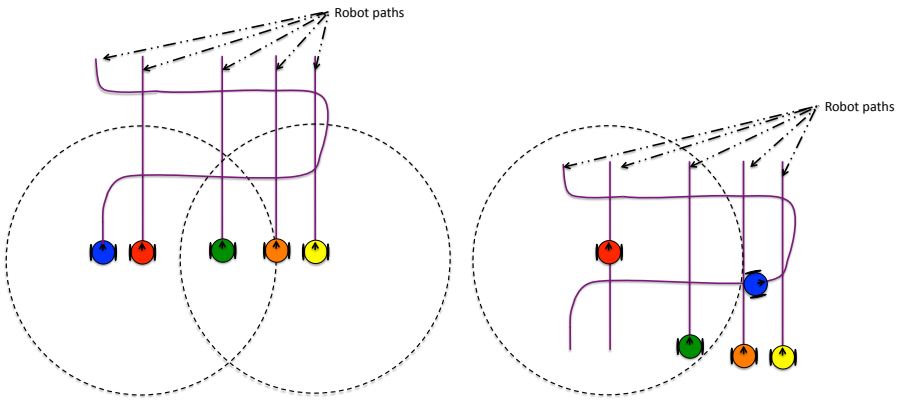

Fig. 3. The paths shown here form an *Infeasible Set*. The dotted circles indicate the communication range of each robot.

with 2 other robots. It is also apparent that these paths are not *Infeasible Paths* as per Definition 2. However, as shown in the figure to the right, at some point during the transit of the red robot, the blue robot moves outside its communication range violating the communication connectivity constraint of $n_{conn} = 2$ at all time steps. Unless, the blue robot has infinite acceleration so that it can traverse the segment of its path that lies outside the communication range of the red robot instantaneously, the problem is infeasible. This is an example of a scenario where the communication constraints are not satisfied due to dynamic bounds.

VI. Conclusion and Future Direction

This paper introduced a communication connectivity constraint formulation for MVPC using MINLP that is robust to communication connectivity failures due to shadowing effects in the environment. An equivalent constraint form was derived that specifies robustness in terms of inter-vehicle communication range. This equivalent form has been validated in simulation and experimentation in our previous work [2], [11]. Furthermore, conditions that affect the feasibility of this problem with respect to communication connectivity were formalized and illustrated. Future work includes incorporating several statistical models that capture the characteristics of indoor and outdoor noisy communication [14] and subsequent experimental validation. As MINLP solution algorithms and numerical solvers continue to improve, we anticipate that this formulation will be applied to a greater number of MVMP problems.

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